Mathematics (AHS)

Formula Booklet

for the Standardised Competence-Oriented
Written School-Leaving Examination

(valid from the academic year 2017/18)

Bundesministerium
Bildung, Wissenschaft
und Forschung

As of 1st September 2017

1 Sets

$\in$	is an element of...
$\notin$	is not an element of...
$\cap$	intersection
$\cup$	union
$\subset$	proper subset
$\subseteq$	subset
$\setminus$	difference ("without")
$\{\ \}$	empty set

Sets of numbers

$\mathbb{N} = \{0, 1, 2, ...\}$	natural numbers
$\mathbb{Z}$	integers
$\mathbb{Q}$	rational numbers
$\mathbb{R}$	real numbers
$\mathbb{C}$	complex numbers
$\mathbb{R}^+$	positive real numbers
$\mathbb{R}_0^+$	positive real numbers including zero

2 Prefixes

tera-	T	10^{12}	deci-	d	10^{-1}
giga-	G	10^{9}	centi-	c	10^{-2}
mega-	M	10^{6}	milli-	m	10^{-3}
kilo-	k	10^{3}	micro-	µ	10^{-6}
hecto-	h	10^{2}	nano-	n	10^{-9}
deca-	da	10^{1}	pico-	p	10^{-12}

3 Powers

Powers with integer exponents

$a \in \mathbb{R}; n \in \mathbb{N}\setminus\{0\}$

$$a^n = \underbrace{a \cdot a \cdot ... \cdot a}_{n \text{ factors}} \qquad a^1 = a$$

$a \in \mathbb{R}\setminus\{0\}; n \in \mathbb{N}\setminus\{0\}$

$$a^0 = 1 \qquad a^{-1} = \frac{1}{a} \qquad a^{-n} = \frac{1}{a^n} = \left(\frac{1}{a}\right)^n$$

Powers with rational exponents (roots)

$a, b \in \mathbb{R}_0^+; n, k \in \mathbb{N}\setminus\{0\}$ where $n \geq 2$

$$a = \sqrt[n]{b} \iff a^n = b \qquad a^{\frac{1}{n}} = \sqrt[n]{a} \qquad a^{\frac{k}{n}} = \sqrt[n]{a^k} \qquad a^{-\frac{k}{n}} = \frac{1}{\sqrt[n]{a^k}} \text{ where } a > 0$$

Calculation rules

$a, b \in \mathbb{R}\setminus\{0\}; r, s \in \mathbb{Z}$ or $a, b \in \mathbb{R}^+; r, s \in \mathbb{Q}$	$a, b \in \mathbb{R}_0^+; m, n, k \in \mathbb{N}\setminus\{0\}$ where $m, n \geq 2$
$a^r \cdot a^s = a^{r+s}$	$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
$\frac{a^r}{a^s} = a^{r-s}$	$\sqrt[n]{a^k} = \left(\sqrt[n]{a}\right)^k$
$(a^r)^s = a^{r \cdot s}$	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ $(b \neq 0)$
$(a \cdot b)^r = a^r \cdot b^r$	$\sqrt[n]{\sqrt[m]{a}} = \sqrt[n \cdot m]{a}$
$\left(\frac{a}{b}\right)^r = \frac{a^r}{b^r}$	

Binomial formulae

$a, b \in \mathbb{R}; n \in \mathbb{N}$

$(a + b)^2 = a^2 + 2 \cdot a \cdot b + b^2$

$(a - b)^2 = a^2 - 2 \cdot a \cdot b + b^2$

$(a + b) \cdot (a - b) = a^2 - b^2$

$(a + b)^n = \sum_{k=0}^{n} \binom{n}{k} \cdot a^{n-k} \cdot b^k$

$(a - b)^n = \sum_{k=0}^{n} (-1)^k \cdot \binom{n}{k} \cdot a^{n-k} \cdot b^k$

4 Logarithms

$a, b, c \in \mathbb{R}^+$ where $a \neq 1; x, r \in \mathbb{R}$

$x = \log_a(b) \Leftrightarrow a^x = b$

$\log_a(b \cdot c) = \log_a(b) + \log_a(c)$ $\quad \log_a\left(\frac{b}{c}\right) = \log_a(b) - \log_a(c)$ $\quad \log_a(b^r) = r \cdot \log_a(b)$

$\log_a(a^x) = x$ $\quad \log_a(a) = 1$ $\quad \log_a(1) = 0$ $\quad \log_a\left(\frac{1}{a}\right) = -1$

natural logarithm (logarithm with base e): $\ln(b) = \log_e(b)$
common logarithm (logarithm with base 10): $\lg(b) = \log_{10}(b)$

5 Quadratic Equations

$p, q \in \mathbb{R}$	$a, b, c \in \mathbb{R}$ where $a \neq 0$
$x^2 + p \cdot x + q = 0$	$a \cdot x^2 + b \cdot x + c = 0$
$x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$	$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a}$

Vieta's Theorem

x_1 and x_2 are the solutions to the equation $x^2 + p \cdot x + q = 0$ if and only if:
$x_1 + x_2 = -p$
$x_1 \cdot x_2 = q$

Linear factorisation:
$x^2 + p \cdot x + q = (x - x_1) \cdot (x - x_2)$

6 Two-Dimensional Shapes

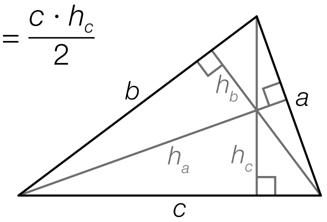
A ... area
u ... perimeter

Triangle

$u = a + b + c$

General triangle

$$A = \frac{a \cdot h_a}{2} = \frac{b \cdot h_b}{2} = \frac{c \cdot h_c}{2}$$

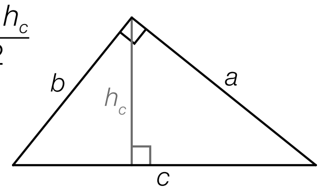


Heron's Formula

$$A = \sqrt{s \cdot (s - a) \cdot (s - b) \cdot (s - c)} \text{ where } s = \frac{a + b + c}{2}$$

Right-angled triangle with hypotenuse *c* and sides *a*, *b*

$$A = \frac{a \cdot b}{2} = \frac{c \cdot h_c}{2}$$



Pythagorean theorem

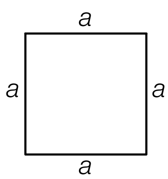
$$a^2 + b^2 = c^2$$

Quadrilateral

Square

$A = a^2$

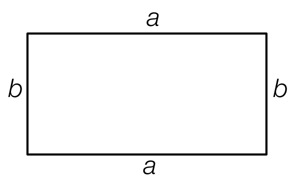
$u = 4 \cdot a$



Rectangle

$A = a \cdot b$

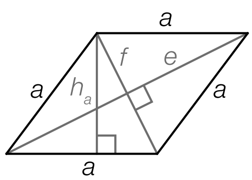
$u = 2 \cdot a + 2 \cdot b$



Rhombus

$A = a \cdot h_a = \frac{e \cdot f}{2}$

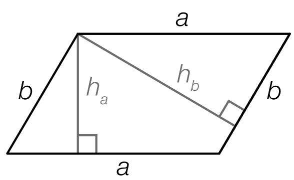
$u = 4 \cdot a$



Parallelogram

$A = a \cdot h_a = b \cdot h_b$

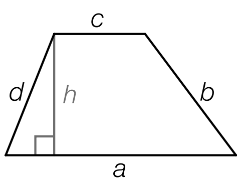
$u = 2 \cdot a + 2 \cdot b$



Trapezium

$A = \frac{(a + c) \cdot h}{2}$

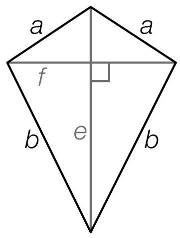
$u = a + b + c + d$



Kite

$A = \frac{e \cdot f}{2}$

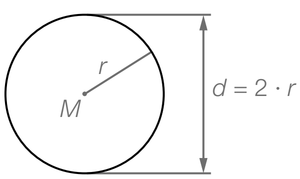
$u = 2 \cdot a + 2 \cdot b$



Circle

$A = \pi \cdot r^2 = \frac{\pi \cdot d^2}{4}$

$u = 2 \cdot \pi \cdot r = \pi \cdot d$

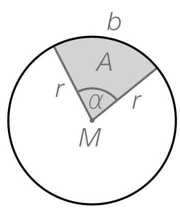


Arc length and sector of a circle

α in degrees (°)

$b = \pi \cdot r \cdot \frac{\alpha}{180°}$

$A = \pi \cdot r^2 \cdot \frac{\alpha}{360°} = \frac{b \cdot r}{2}$



7 Solids

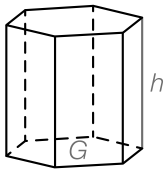
V ... volume
O ... surface area
G ... area of the base
M ... lateral surface area
u_G ... perimeter of the base

Prism

$V = G \cdot h$

$M = u_G \cdot h$

$O = 2 \cdot G + M$

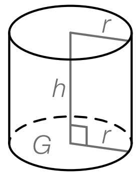


Cylinder

$V = G \cdot h$

$M = u_G \cdot h$

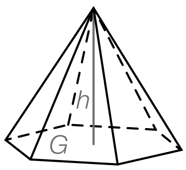
$O = 2 \cdot G + M$



Pyramid

$V = \frac{G \cdot h}{3}$

$O = G + M$



Cone

$V = \frac{G \cdot h}{3}$

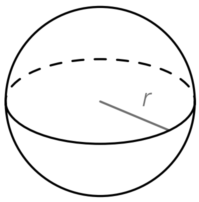
$M = \pi \cdot r \cdot s$

$O = G + M$

Sphere

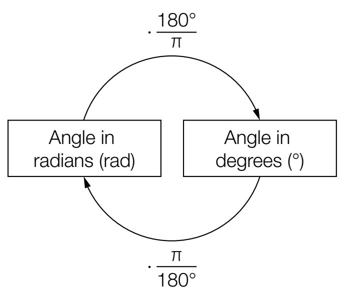
$V = \frac{4}{3} \cdot \pi \cdot r^3$

$O = 4 \cdot \pi \cdot r^2$



8 Trigonometry

Converting between degrees and radians

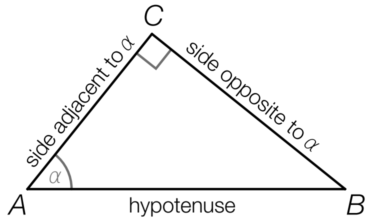


Right-angled triangle trigonometry

Sine: $\sin(\alpha) = \frac{\text{side opposite to } \alpha}{\text{hypotenuse}}$

Cosine: $\cos(\alpha) = \frac{\text{side adjacent to } \alpha}{\text{hypotenuse}}$

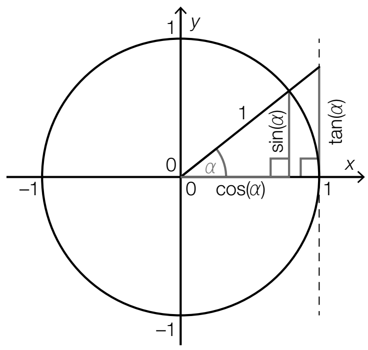
Tangent: $\tan(\alpha) = \frac{\text{side opposite to } \alpha}{\text{side adjacent to } \alpha}$



Unit circle trigonometry

$\sin^2(\alpha) + \cos^2(\alpha) = 1$

$\tan(\alpha) = \dfrac{\sin(\alpha)}{\cos(\alpha)}$ for $\cos(\alpha) \neq 0$



9 Vectors

P, *Q* ... points

Vectors in $\mathbb{R}^2$

Arrow from *P* to *Q*:

$P = (p_1|p_2)$, $Q = (q_1|q_2)$

$\overrightarrow{PQ} = \begin{pmatrix} q_1 - p_1 \\ q_2 - p_2 \end{pmatrix}$

Vectors in $\mathbb{R}^n$

Arrow from *P* to *Q*:

$P = (p_1|p_2|...|p_n)$, $Q = (q_1|q_2|...|q_n)$

$\overrightarrow{PQ} = \begin{pmatrix} q_1 - p_1 \\ q_2 - p_2 \\ \vdots \\ q_n - p_n \end{pmatrix}$

Calculation rules in $\mathbb{R}^2$

$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$, $\vec{a} \pm \vec{b} = \begin{pmatrix} a_1 \pm b_1 \\ a_2 \pm b_2 \end{pmatrix}$

$k \cdot \vec{a} = k \cdot \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} k \cdot a_1 \\ k \cdot a_2 \end{pmatrix}$ where $k \in \mathbb{R}$

Calculation rules in $\mathbb{R}^n$

$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}$, $\vec{b} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$, $\vec{a} \pm \vec{b} = \begin{pmatrix} a_1 \pm b_1 \\ a_2 \pm b_2 \\ \vdots \\ a_n \pm b_n \end{pmatrix}$

$k \cdot \vec{a} = k \cdot \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} = \begin{pmatrix} k \cdot a_1 \\ k \cdot a_2 \\ \vdots \\ k \cdot a_n \end{pmatrix}$ where $k \in \mathbb{R}$

Scalar product in $\mathbb{R}^2$

$\vec{a} \cdot \vec{b} = a_1 \cdot b_1 + a_2 \cdot b_2$

Scalar product in $\mathbb{R}^n$

$\vec{a} \cdot \vec{b} = a_1 \cdot b_1 + a_2 \cdot b_2 + ... + a_n \cdot b_n$

Absolute value (length) of a vector in $\mathbb{R}^2$

$|\vec{a}| = \sqrt{a_1^2 + a_2^2}$

Absolute value (length) of a vector in $\mathbb{R}^n$

$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + ... + a_n^2}$

Vector perpendicular to $\vec{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ in $\mathbb{R}^2$

$\vec{n} = k \cdot \begin{pmatrix} -a_2 \\ a_1 \end{pmatrix}$ where $k \in \mathbb{R}\backslash\{0\}$ and $|\vec{a}| \neq 0$

Angle φ between $\vec{a}$ and $\vec{b}$ in $\mathbb{R}^2$ and $\mathbb{R}^3$

$\cos(\varphi) = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}$ where $|\vec{a}| \neq 0$; $|\vec{b}| \neq 0$

Criterion for two vectors to be perpendicular in $\mathbb{R}^2$ and $\mathbb{R}^3$

$\vec{a} \cdot \vec{b} = 0 \iff \vec{a} \perp \vec{b}$ where $|\vec{a}| \neq 0$; $|\vec{b}| \neq 0$

Criterion for two vectors to be parallel in $\mathbb{R}^2$ and $\mathbb{R}^3$

$\vec{a} \parallel \vec{b} \iff \vec{a} = k \cdot \vec{b}$ where $k \in \mathbb{R}\backslash\{0\}$ and $|\vec{a}| \neq 0$; $|\vec{b}| \neq 0$

10 Straight Lines

g ... line	$\vec{g}$ ... a direction vector for the line g
	$\vec{n}$ ... a vector perpendicular to the line g
	X, P ... points on the line g
	m ... gradient of the line g
	α ... angle of slope of the line g
	$a, b, c, k \in \mathbb{R}$

Vector equation of a line g in $\mathbb{R}^2$ and $\mathbb{R}^3$

$g: X = P + t \cdot \vec{g}$ where $t \in \mathbb{R}$

Equation of a line g in $\mathbb{R}^2$

the explicit equation of a line: $g: y = m \cdot x + c$ where $m = \tan(\alpha)$

a general equation of a line: $g: a \cdot x + b \cdot y = c$

a normal vector representation: $g: \vec{n} \cdot X = \vec{n} \cdot P$

(for the general equation and the normal vector representation) where $\vec{n} \parallel \begin{pmatrix} a \\ b \end{pmatrix}$ and $\begin{pmatrix} a \\ b \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

11 Rates of Change

For a real function f defined over an interval $[a, b]$:

Absolute change of f in $[a, b]$

$f(b) - f(a)$

Relative (percentage) change of f in $[a, b]$

$\frac{f(b) - f(a)}{f(a)}$ where $f(a) \neq 0$

Difference quotient (average rate of change) of f in $[a, b]$ or $[x, x + \Delta x]$

$\frac{f(b) - f(a)}{b - a}$ or $\frac{f(x + \Delta x) - f(x)}{\Delta x}$ where $b \neq a$ and $\Delta x \neq 0$

Differential quotient (instantaneous rate of change) of f at the point x

$f'(x) = \lim\limits_{x_1 \to x} \frac{f(x_1) - f(x)}{x_1 - x}$ or $f'(x) = \lim\limits_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$

12 Differentiation and Integration

f, g, h ... functions that are differentiable over $\mathbb{R}$ or over a defined interval

f' ... first derivative of f	F ... antiderivative of f
g' ... first derivative of g	G ... antiderivative of g
h' ... first derivative of h	H ... antiderivative of h

$C, k, q \in \mathbb{R}$; $a \in \mathbb{R}^+\setminus\{1\}$

Indefinite integral

$$\int f(x)\,\mathrm{d}x = F(x) + C \quad \text{where } F' = f$$

Definite integral

$$\int_a^b f(x)\,\mathrm{d}x = F(x)\Big|_a^b = F(b) - F(a)$$

Function	Derivative	Antiderivative
$f(x) = k$	$f'(x) = 0$	$F(x) = k \cdot x$
$f(x) = x^q$	$f'(x) = q \cdot x^{q-1}$	$F(x) = \frac{x^{q+1}}{q+1}$ where $q \neq -1$ $F(x) = \ln(\lvert x \rvert)$ where $q = -1$
$f(x) = e^x$	$f'(x) = e^x$	$F(x) = e^x$
$f(x) = a^x$	$f'(x) = \ln(a) \cdot a^x$	$F(x) = \frac{a^x}{\ln(a)}$
$f(x) = \sin(x)$	$f'(x) = \cos(x)$	$F(x) = -\cos(x)$
$f(x) = \cos(x)$	$f'(x) = -\sin(x)$	$F(x) = \sin(x)$
$g(x) = k \cdot f(x)$	$g'(x) = k \cdot f'(x)$	$G(x) = k \cdot F(x)$
$h(x) = f(x) \pm g(x)$	$h'(x) = f'(x) \pm g'(x)$	$H(x) = F(x) \pm G(x)$
$g(x) = f(k \cdot x)$	$g'(x) = k \cdot f'(k \cdot x)$	$G(x) = \frac{1}{k} \cdot F(k \cdot x)$

13 Statistics

$x_1, x_2, \dots, x_n$ ... a list of n real numbers

$x_{(1)} \leq x_{(2)} \leq \dots \leq x_{(n)}$ ... ordered list of n values

Arithmetic mean

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} = \frac{1}{n} \cdot \sum_{i=1}^{n} x_i$$

Median

$$\tilde{x} = \begin{cases} x_{\left(\frac{n+1}{2}\right)} & \text{... when } n \text{ is odd} \\ \frac{1}{2} \cdot \left(x_{\left(\frac{n}{2}\right)} + x_{\left(\frac{n}{2}+1\right)}\right) & \text{... when } n \text{ is even} \end{cases}$$

Measures of spread

s^2 ... (empirical) variance of a sample

s ... (empirical) standard deviation of a sample

$$s^2 = \frac{1}{n} \cdot \sum_{i=1}^{n} (x_i - \bar{x})^2$$

$$s = \sqrt{\frac{1}{n} \cdot \sum_{i=1}^{n} (x_i - \bar{x})^2}$$

If the variance of a population should be estimated using a sample of size n:

$$s_{n-1}^2 = \frac{1}{n-1} \cdot \sum_{i=1}^{n} (x_i - \bar{x})^2$$

$$s_{n-1} = \sqrt{\frac{1}{n-1} \cdot \sum_{i=1}^{n} (x_i - \bar{x})^2}$$

14 Probability

$n \in \mathbb{N}\setminus\{0\}$; $k \in \mathbb{N}$ where $k \leq n$
A, B ... events
$\neg A$ or $\overline{A}$ ... complementary event of A
$A \wedge B$ or $A \cap B$ ... A and B (the event A and the event B both occur)
$A \vee B$ or $A \cup B$ ... A or B (at least one of the two events A or B occurs)
$P(A)$ ... probability of event A occurring
$P(A|B)$ ... probability of event A occurring given that B has occurred (conditional probability)

Factorial

$n! = n \cdot (n-1) \cdot ... \cdot 1 \qquad 0! = 1 \qquad 1! = 1$

Binomial coefficient

$\binom{n}{k} = \frac{n!}{k! \cdot (n-k)!}$

Probability for a Laplace experiment

$$P(A) = \frac{\text{number of successful outcomes for } A}{\text{number of possible outcomes}}$$

Elementary rules

$P(\neg A) = 1 - P(A)$
$P(A \wedge B) = P(A) \cdot P(B|A) = P(B) \cdot P(A|B)$
$P(A \wedge B) = P(A) \cdot P(B)$ ... if A and B are (stochastically) independent of one another
$P(A \vee B) = P(A) + P(B) - P(A \wedge B)$
$P(A \vee B) = P(A) + P(B)$ ... if A and B are mutually exclusive

Expectation value μ of a discrete random variable X with values $x_1, x_2, ..., x_n$

$$\mu = E(X) = x_1 \cdot P(X = x_1) + x_2 \cdot P(X = x_2) + ... + x_n \cdot P(X = x_n) = \sum_{i=1}^{n} x_i \cdot P(X = x_i)$$

Variance σ^2 of a discrete random variable X with values $x_1, x_2, ..., x_n$

$$\sigma^2 = V(X) = \sum_{i=1}^{n} (x_i - \mu)^2 \cdot P(X = x_i)$$

Standard deviation σ

$$\sigma = \sqrt{V(X)}$$

Binomial distribution

$n \in \mathbb{N}\setminus\{0\}$; $k \in \mathbb{N}$; $p \in \mathbb{R}$ where $k \leq n$ and $0 \leq p \leq 1$

The random variable X is binomially distributed with parameters n and p

$$P(X = k) = \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k}$$

$$E(X) = \mu = n \cdot p$$

$$V(X) = \sigma^2 = n \cdot p \cdot (1-p)$$

Normal distribution

$\mu, \sigma \in \mathbb{R}$ where $\sigma > 0$
f ... probability density function
φ ... probability density function of the standard normal distribution
ϕ ... cumulative density function of the standard normal distribution

Normal distribution $N(\mu; \sigma^2)$: The random variable X is normally distributed with expectation value (μ), standard deviation (σ) and variance (σ^2)

$$P(X \leq x_1) = \int_{-\infty}^{x_1} f(x)\,\mathrm{d}x = \int_{-\infty}^{x_1} \frac{1}{\sigma \cdot \sqrt{2 \cdot \pi}} \cdot e^{-\frac{1}{2} \cdot \left(\frac{x-\mu}{\sigma}\right)^2} \mathrm{d}x$$

Probabilities for standard deviation bands

$P(\mu - \sigma \leq X \leq \mu + \sigma) \approx 0.683$
$P(\mu - 2 \cdot \sigma \leq X \leq \mu + 2 \cdot \sigma) \approx 0.954$
$P(\mu - 3 \cdot \sigma \leq X \leq \mu + 3 \cdot \sigma) \approx 0.997$

Standard normal distribution $N(0, 1)$

$$z = \frac{x - \mu}{\sigma}$$

$$\phi(z) = P(Z \leq z) = \int_{-\infty}^{z} \varphi(x)\,\mathrm{d}x = \frac{1}{\sqrt{2 \cdot \pi}} \cdot \int_{-\infty}^{z} e^{-\frac{x^2}{2}} \mathrm{d}x$$

$$\phi(-z) = 1 - \phi(z)$$

$$P(-z \leq Z \leq z) = 2 \cdot \phi(z) - 1$$

$P(-z \leq Z \leq z)$	$= 90$ %	$= 95$ %	$= 99$ %
z	≈ 1.645	≈ 1.960	≈ 2.576

Confidence interval

h ... relative frequency in a sample
p ... unknown relative proportion of the population
γ ... confidence level

γ-confidence interval for p (the values of p for which the value h is contained in the given range with probability γ):

$$\left[h - z \cdot \sqrt{\frac{h \cdot (1-h)}{n}}; h + z \cdot \sqrt{\frac{h \cdot (1-h)}{n}}\right], \text{ where for } z\text{: } \gamma = 2 \cdot \phi(z) - 1$$

15 Units of Measurement

Quantity	Unit	Symbol	Relationship
Temperature	degrees Celsius or kelvin	°C K	$\Delta t = \Delta T$
Frequency	hertz	Hz	$1\ Hz = 1\ s^{-1}$
Energy, work done, amount of heat	joules	J	$1\ J = 1\ kg \cdot m^2 \cdot s^{-2}$
Force	newtons	N	$1\ N = 1\ kg \cdot m \cdot s^{-2}$
Torque	newton metres	N · m	$1\ N \cdot m = 1\ kg \cdot m^2 \cdot s^{-2}$
Electric resistance	ohms	Ω	$1\ \Omega = 1\ V \cdot A^{-1} =$ $1\ kg \cdot m^2 \cdot A^{-2} \cdot s^{-3}$
Pressure	pascals	Pa	$1\ Pa = 1\ N \cdot m^{-2} =$ $1\ kg \cdot m^{-1} \cdot s^{-2}$
Electric current	amperes	A	$1\ A = 1\ C \cdot s^{-1}$
Potential difference	volts	V	$1\ V = 1 \cdot J \cdot C^{-1} =$ $1\ kg \cdot m^2 \cdot A^{-1} \cdot s^{-3}$
Power	watts	W	$1\ W = 1\ J \cdot s^{-1} =$ $1\ kg \cdot m^2 \cdot s^{-3}$

16 Physical Quantities and Definitions

Density	$\varrho = \frac{m}{V}$		
Power	$P = \frac{\Delta E}{\Delta t}$	$P = \frac{\Delta W}{\Delta t}$	$P = \frac{\mathrm{d}W(t)}{\mathrm{d}t}$
Force	$F = m \cdot a$		
Work done	$W = F \cdot s$ $W = \int F(s)\,\mathrm{d}s$	$F = \frac{\mathrm{d}W}{\mathrm{d}s}$	
Kinetic energy	$E_{\mathrm{kin}} = \frac{1}{2} \cdot m \cdot v^2$		
Potential energy	$E_{\mathrm{pot}} = m \cdot g \cdot h$		
Uniform linear motion	$v = \frac{s}{t}$	$v = \frac{\mathrm{d}s}{\mathrm{d}t}$	$v(t) = s'(t) = \frac{\mathrm{d}s}{\mathrm{d}t}$
Uniform acceleration	$v = a \cdot t + v_0$	$a = \frac{\mathrm{d}v}{\mathrm{d}t}$	$a(t) = v'(t) = \frac{\mathrm{d}v}{\mathrm{d}t} = s''(t) = \frac{\mathrm{d}^2 s}{\mathrm{d}t^2}$

17 Financial Mathematics

Compound interest calculation

K_0 ... initial investment
K_n ... final capital
p ... annual percentage rate of interest

$K_n = K_0 \cdot (1 + i)^n$ where $i = \frac{p}{100}$

Cost-of-production theory of value

x ... amount produced, offered, required or sold ($x \geq 0$)

Variable costs	$K_{\mathrm{v}}(x)$
Fixed costs	K_{f}
(Total) costs	$K(x) = K_{\mathrm{v}}(x) + K_{\mathrm{f}}$
Marginal costs	$K'(x)$
Demand price	$p(x)$
Revenue/income	$E(x) = p(x) \cdot x$
Marginal revenue	$E'(x)$
Profit	$G(x) = E(x) - K(x)$
Marginal profit	$G'(x)$
Break-even point	$E(x) = K(x)$ ... at the (first) zero of the profit function